**91029**

**B.Sc. 1st Semester (Hons.) Examination,**

**November–2014**

**PHYSICS**

**Paper–Phy–104**

**Mathematics–I**

*Time allowed : 3 hours ]* *[ Maximum marks : 40*

***Note :*** *Attempt **five** questions in all, selecting at least **two** questions from each section. All questions carry equal marks.*

## Section–I

**1.** (a) Prove that every convergent sequence is bounded but not conversely. 4

(b) State and prove Cauchy's first theorem on limits. 4

**2.** (a) Prove that $\lim_{n\to\infty}\left(\frac{n^n}{n!}\right)^{1/n} = e$ 4

(b) Prove that a monotonically increasing sequence $< a_n >$ which is bounded above converges to its least upper bount. 4

3. (a) Prove that every Cauchy's sequence is bounded. Is the converse true ? Show by an example. 4

(b) Discuss the convergence of the sequence $< a_n >$ where $a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + ... + \frac{1}{3^n}$. 4

4. (a) Prove that a positive term series either converges or diverges to $+\infty$. 4

(b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$ converges to $1/4$ 4

## Section–II

5. (a) If $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ are two series of positive terms and $\sum_{n=1}^{\infty} u_n$ is divergent and there is positive constant k such that $u_n \leq k v_n$, $\forall$ n, then show that $\sum_{n=1}^{\infty} v_n$ is also divergent. 4

(b) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty} \left( \sqrt{n^3 + 1} - \sqrt{n^3} \right)$$ 4

6. (a) Test the convergence of the series :

$$\frac{1}{1.2.3}+\frac{x}{4.5.6}+\frac{x^2}{7.8.9}+\ldots\ldots,\ x>0$$ 4

(b) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n x^n,\quad x>0.$$ 4

7. (a) State and prove Cauchy's integral test. 4

(b) Test the convergence of the series

$$x^2+\frac{2^2}{3.4}x^4+\frac{2^2.4^2}{3.4.5.6}x^6+\frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^8+\ldots.,\ x>0.$$

4

8. (a) Show that every absolutely convergent series is convergent. Is the converse true ? Show by an example. 4

(b) Test the convergence and absolute convergence of the series :

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots$$ 4